Challenging Trigonometric Problems AIME

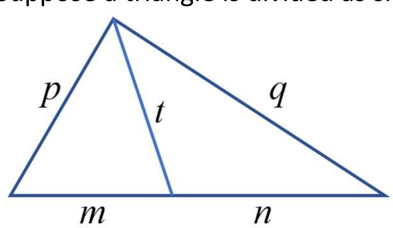
1. Given that ΔABC is inside a circle with radius "R", with all three vertices on the circumference. Prove the following:

   i) $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = 2R$ ii) $Area\ \Delta ABC = \dfrac{abc}{4R}$

   iii) $Area\ \Delta ABC = 2R^2 \sin a \sin b \sin C$

   iv) $Area\ \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$ [yes, try to prove Heron's formula]

2. Suppose a triangle is divided as shown, with the sides given:



Stewart's theorem states that: $np^2 + mq^2 = (m+n)(t^2 + mn)$ Prove Stewart's theorem

3. Given that $(1+\sin t)(1+\cos t)=\frac{5}{4}$ and $(1-\sin t)(1-\cos t)=\frac{m}{n}-\sqrt{k}$, where “k”, “m”, and “n” are positive integers with “m” and “n” relatively prime, find $k+m+n$ (1995 AIME)

4. Let $x=\frac{\sum_{n=1}^{44}\cos n}{\sum_{n=1}^{44}\sin n}$. What is the greatest integer that not exceed $100x$? (1997 aime)

5. Let “x” and “y’ be real numbers such that $\frac{\sin x}{\sin y}=3$ and $\frac{\cos x}{\cos y}=\frac{1}{2}$. Find the value of $\frac{\sin 2x}{\sin 2y}+\frac{\cos 2x}{\cos 2y}$ as a fraction.

6. Find the least positive integer “n” such that:

$$\frac{1}{\sin 45^\circ \sin 46^\circ}+\frac{1}{\sin 47^\circ \sin 48^\circ}+\ldots\frac{1}{\sin 133^\circ \sin 134^\circ}=\frac{1}{\sin n^\circ}$$ (aime 2000 #15)